Reply to "Comment on 'Analog of Planck's formula and effective temperature in classical statistical mechanics far from equilibrium' "

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We emphasize that the interpretation of Planck's law under discussion, which is given within the framework of classical mechanics and is based on estimates for the exchanges of energy in models of molecular dynamics, actually makes reference to two well-separated time scales. On a very short time scale a state of metaequilibrium would be established such that the exchanges of energy with a measurement instrument would be described by a Planck-like law, while equipartition of energy would be established over a much larger time scale, as occurs, for example, in the case of glasses.

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Let us recall the main issue of our paper [1], which was the following one. On one hand, we recalled a suggested interpretation that Einstein had given of Planck's law in terms of fluctuations of energy, namely, how he had shown that Planck's law is essentially equivalent to a certain functional relation between variance of energy and mean energy. On the other hand, we pointed out that a functional relation of that very form comes out of the analytical formulas for the exchanges of energy between vibrations and centers of mass in atomic collisions that can be found with the known procedure of Landau and Teller. This simple remark looked interesting to us and worthy of publication.

Cavalleri and Cesaroni argue [2] that such an interpretation of Planck's law would not be suited to describing certain realistic situations. The core of the argument makes reference to an experiment in which one considers the black body radiation in equilibrium inside a cavity at a certain temperature and the temperature is quickly lowered, for example, to half the original value. Apparently, a new equilibrium is rapidly attained and this fact would be in contrast with the above mentioned interpretation of Planck's law because it makes reference (see below) to situations of metaequilibrium, where the final equilibrium is reached after huge times.

It seems to us that there is a misunderstanding because the scenario to which we are making reference is actually involving two sharply separated time scales. Perhaps this was not emphasized clearly enough in our paper and so we willingly take this opportunity to better explain it. Notice that our considerations are of a completely qualitative character because we have no model available for the black body and we are actually referring (here, as in our paper) to models of molecular dynamics, such as the Landau-Teller model of molecular collisions or the Fermi-Pasta-Ulam (FPU) model.

The point we want to clarify requires that a few preliminary words be added concerning the exchanges of energy between internal vibrations and centers of mass in atomic collisions that are estimated, for example, in the paradigmatic model of Landau and Teller. The key point is that the energy exchanges turn out to be exponentially small as the frequency of the internal vibrations is increased or the temperature is decreased. This is the reason why, according to classical mechanics, in order for the internal degrees of freedom the "final" relaxation to a Maxwell-Boltzmann equilibrium might require a huge time, even of geological orders of magnitude, while on a rather short time scale one would have a collapse to a kind of metaequilibrium state, as in the familiar case of glasses. The question is now whether there exists some kind of thermodynamics even in such situations of metaequilibrium. This is actually the main point of our paper, because we have shown that, if one looks at the "exchanged energy," an average over few collisions (this is the point that perhaps was not emphasized enough) is enough to produce between variance and mean a functional relation which has the form indicated by Einstein as essentially equivalent to Planck's law.

Thus, if such an indication is taken seriously, a scenario emerges in which a distribution of Planck's type is quickly established for the exchanged energy, which the quantity actually detected by measurement instruments. This would describe the thermodynamics suited for a kind of metaequilibrium state, which would later evolve, over an extremely larger time scale, to a final Maxwell-Boltzmann distribution. Such a scenario appears to be supported by some recent results on the FPU problem [3] and on the Landau-Teller model [4] (see also Refs. [5,6]). In such a way, the objection of Cavalleri and Cesaroni should be overcome, at least at a qualitative level.

As a final comment, we take this opportunity to mention that there exists (as we recently came to realize) an improvement of the main argument at the basis of our paper. This might be of interest for the present discussion, because it apparently gives support to the idea that Planck's law might be interpreted, in the framework of classical mechanics, as describing an off-equilibrium state. The improvement amounts to replacing the relation $dU/d\beta = -\sigma_E^2$ between mean and variance of energy, which was at the basis of Einstein's approach to Planck's law and has a static character, by the analogous relation that holds according to the fluctuation dissipation theorem, and makes instead reference to an off-equilibrium situation. This fact was briefly illustrated in a recent review paper of ours [7], where the relevance of a sharp separation between the two above mentioned time scales was also emphasized.

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